The Development of Mathematics Education Software in China

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Abstract

The value of dynamic geometry software in mathematics education has been widely recognized for several decades. However, the exam-oriented approach to education takes precedence in Mainland China. Therefore, most of the mathematics teachers in Chinese secondary schools are faced with a heavy burden of work, and have little time to study dynamic geometry software, which does not have an immediately noticeable improvement in students' exam performance.

The author of this paper has been engaged in designing and developing mathematics education software and resources for more than 10 years, and has served as a director and advisor for the in-service mathematics teachers on dynamic geometry software for six years. According to his experience, he gives a description of the current state of the Information Technology (IT) based mathematics education in China. Then he presents his viewpoints on why dynamic geometry software was not widely used.

Details on SuperSketchPad (SSP), a dynamic mathematics software package designed and developed for Chinese mathematics education, and its use in China will be introduced in this paper. Based on the findings of the experimental research on using SuperSketchpad in mathematics education, the author concludes that designing and developing the mathematics education software for a region should take the various needs of the teachers in that region into serious consideration.

1. The current state of the IT-based mathematics education in China

As we all know, the geometrical attributes of figures constructed by dynamic geometry software remain constant when the figures change. With dynamic geometry software, students can do constructions and measurements, manipulate the figures to explore the relationships, make and test conjectures, and conduct further explorations. Students can also plot the curves of functions with parameters, then study the relationships between the functions and parameters, and so on. The value of dynamic geometry software in mathematics education has been widely recognized for several decades. For example, the Geometer's Sketchpad (GSP, see [1]) is popularly embraced by mathematics teachers from many countries. It was introduced into China in 1996. Several other dynamic geometry software packages, such as Cabri-Geometry and GeoGebra (see [2] and [3]), are also known by a lot of mathematics teachers in China.

However, the use of dynamic geometry software in mathematics education in most of the regions of China has not reached the expectation of mathematics educators and that of the Ministry of Education.

Large amounts of data have shown that:

(1) Most of Chinese mathematics teachers prefer the traditional chalk-blackboard method to using interactive geometry software. When teachers are invited to teach exemplary model lessons, they present the prepared materials using PowerPoint rather than dynamic geometry software.

(2) In economic-advanced regions of China, the frequency of using computers in teachers' daily teaching activities is higher, but few teachers choose to use dynamic geometry software.

(3) GSP is the most well-known dynamic geometry software in China. Websites [4] and [5] have attracted many experienced users of GSP to exchange ideas through the Internet. However, there are lots of difficulties for general mathematics teachers when they develop the learning activities with GSP for daily teaching purposes. They usually give up using GSP if they cannot find the existing materials that they want.

(4) The main purpose of mathematics teachers using dynamic geometry software is to display dynamic graphs. Few teachers are able to use it in facilitating students' mathematical explorations.

We think the following are the main reasons for the aforementioned situations:

(1) Although we have achieved a lot in mathematics education, we have also experienced a number of problems. The biggest one among them is the exam-oriented education. The exam-first education is closely related to the entrance examinations. It primarily teaches the content that is included or will possibly be included in the entrance exams. Then students are required to practice repeatedly or they are likely to fail in the exams. What is assessed is what is taught. So most of the mathematics teachers in secondary schools are faced with a heavy burden of work and have little time to study dynamic geometry software, which does not have an immediately noticeable improvement in student exam performance, even if the value of the software in mathematics education is widely recognized by many educators in China.

(2) Mathematics activities usually involve drawing, plotting, measuring, calculating (as shown in Figure 1), deducing (as shown in Figure 2), proving, writing, and so on. Thus mathematics teachers must spend time on learning some computer algebra system besides dynamic geometry software.

5^{3}
$=125;$
50^{3}
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$=243;$
3^{50}
$=717897987691852588770249;$
3^{500}
$=3636029179586993684238526707954331911802338502600162304034603583258060019158389548419850826297938878330817970253440385575285593151701306614299243091656202578002177124784764345012534283656581320997259037159015257872800838599013979537761000 1;$

Figure 1: *calculating the value of powers*

$(a+2b)^{2}$
$=a^{2}+4ab+4b^{2};$
$(a+2b)^{3}$
$=a^{3}+6a^{2}b+12ab^{2}+8b^{3};$
$(a+2b)^{4}$
$=a^{4}+8a^{3}b+24a^{2}b^{2}+32ab^{3}+16b^{4};$
$(a+2b)^{5}$
$=a^{5}+10a^{4}b+40a^{3}b^{2}+80a^{2}b^{3}+80ab^{4}+32b^{5};$
$(a+2b)^{6}$
$=a^{6}+12a^{5}b+60a^{4}b^{2}+160a^{3}b^{3}+240a^{2}b^{4}+192ab^{5}+64b^{6};$
$(a+2b)^{7}$
$=a^{7}+14a^{6}b+84a^{5}b^{2}+280a^{4}b^{3}+560a^{3}b^{4}+672a^{2}b^{5}+448ab^{6}+128b^{7};$

Figure 2: *expanding the expressions*

Recently probability and statistics have been included in the new mathematics curriculum for secondary schools in China. To help students build the basic concepts of probability and statistics, it is necessary to do simulation experiments on computers such as the one shown in Figure 3. So mathematics teachers also need to master the dynamic simulation software.

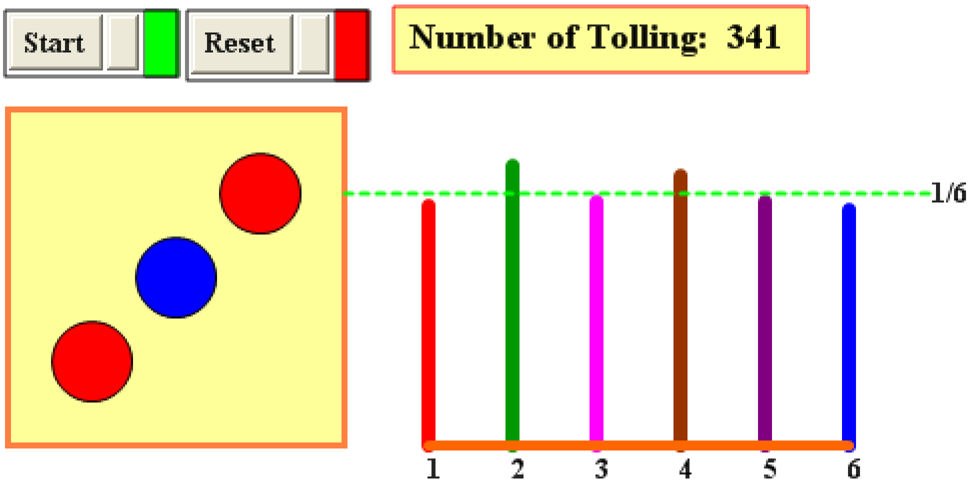

Figure 3: *simulation experiment*

(3) The majority of mathematics teachers would like to master at least one of the mathematics education software packages if they can benefit from it in reducing their workload and improving their work efficiency. In many aspects (such as dynamic movement, dynamic measurement, dynamic transformations, and so on) the dynamic geometry software is much more efficient than human beings' hand drawing. But the current dynamic software does have some limitations. The following are some examples:

①It is not so convenient for dynamic geometry software to construct some basic geometric figures such as isosceles triangle, parallelogram, rectangle, and trapezoid, which frequently appear in the teaching and learning of mathematics. Teachers and students should not spend too much time on the routine tasks of constructing these basic figures.

②More than 30 steps have to be finished if we want to construct the three perpendicular segments shown in Figure 4 (see [6]). Is wound be better for dynamic geometry software to include the capability of drawing these perpendicular segments and their feet directly just like constructing the midpoint of a segment.

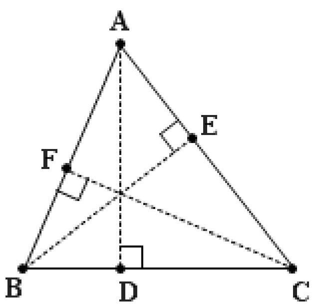

Figure 4: *the three perpendicular segments of a triangle*

③It is unnecessary for students to construct the circumcenter, centroid, orthocenter, and incenter of a triangle step by step after they are familiar with these geometric objects and their construction procedures. If the constructions of these centers are added into the Construct menu, students and teachers will save a lot of time from doing mechanical and uncreative work and thus concentrate on exploring and understanding the related, important mathematics ideas.

④Some dynamic geometry software packages have the "custom tools" feature that allows users to construct their own tools such as the Circumcircle of a Triangle, but general users have difficulties in constructing such "custom tools" themselves.

⑤Teachers usually highlight pairs of corresponding objects in two congruent or similar shapes with simple marks as shown in Figure 5. This gives much help for teaching and learning mathematics easily. However, doing so is very time-consuming for most of dynamic geometry software compared to the chalk-blackboard or paper-pencil approach.

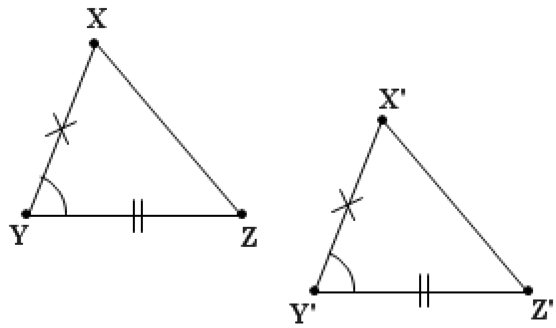

Figure 5: *marks of two congruent triangles*

⑥In many activities, users usually hope to plot the movable coordinate points directly, but it is not so simple as we expect with the current dynamic geometry software. The following is an example about coordinates with parameters:

Segment AB with endpoints A and B on x-axis and y-axis respectively has the fixed length 5. What is the shape of the region scanned by AB when point A slides on x-axis? As we know, the coordinate points A (5*cos(t),0) and B (0,5*sin(t)) meet the requirements of segment AB. But with the current dynamic software, it takes us more than 10 steps to construct point A and B (See [7]).

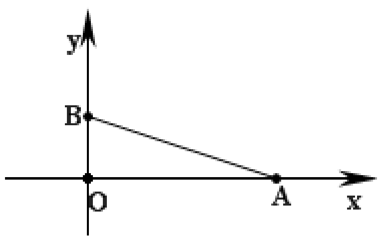

Figure 6: *segment AB has the fixed length 5 which endpoints are movable*

⑦Conic sections cannot be plotted directly with many dynamic geometry software packages, and a conic section only appears as the locus of some point. This situation has many limitations for furthermore explorations. For example, the intersection point(s) of an ellipse and a straight line (P and Q in Figure 7) cannot be plotted directly (see [8]).

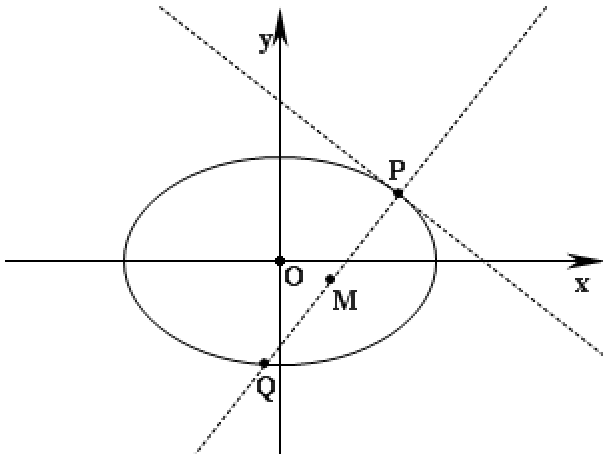

Figure 7: *intersection points of ellipse and straight line*

Knowledge about conic sections is one of the key content topics of mathematics curriculum in secondary schools. So if the usual methods of constructing conic sections are added into the Construct menu, teachers and students will have more time to focus on the important properties and relationships of these curves. Although conic sections can be plotted directly through entering commands in GeoGebra, teachers and students prefer visualized operations to algebraic input.

In sum, teachers have many things to do in their limited time. This is why the dynamic geometry software is not widely used and why mathematics teachers are lack of enthusiasm in using computers in their daily activities. Without teachers' guidance, students cannot use computers effectively in their mathematics learning. So most students in China don’t get the opportunity to benefit from the IT.

2. What kind of software is suitable for Chinese mathematics education?

Both teachers and students should benefit from digital technology. Since the exam-oriented education will not disappear in the near future, we should concentrate on improving the mathematics education software itself.

Dynamic geometry software can enhance mathematics teaching and learning, and makes mathematics fun. At the same time, it should be more user-friendly, and can be used in more mathematics content areas. This will save teachers’ time and reduce their workload so that it becomes possible for them to use mathematics education software in their daily activities.

How can such mathematics education software be developed?

In their classrooms teachers need to do many kinds of work such as plotting, computing, writing, and so on within a fixed time. So the developers of mathematics education software must take these factors into consideration.

(1) The developers should respect the tradition of mathematics education, and analyse the kinds of work faced by mathematics teachers in their daily activities. In simple terms, computer constructions of mathematical obejcts should be at least faster than human beings' hand manipulations.

(2) To meet the standards of teaching, learning and research in mathematics at a certain level (e.g., high school level), the mathematics education software should cover most of the mathematical content at that level.

(3) The mathematics education software should be easy to use so that teachers can develop their own teaching materials after they learn about the main functions and fundamental operations of the software.

An excellent software package should allow users to attain what they need without too many techniques, especially for mathematics teachers in China who have a heavy workload.

3. The Development and Use of a New Mathematics Education Software Package in China

Since 1996, Professor Zhang Jingzhong, a Chinese mathematician, and his team members have been working on designing and developing mathematics education software. In the past decade, they analysed the various needs of mathematics teachers from all over China and accepted their advices and suggestions on the software design.

Their final research achievement is SuperSketchPad (SSP) of which the former version was Mathematical Laboratory published in 1998 and presented at the 4th Asian Technology Conference on Mathematics.

3.1. What is SSP?

SSP is a dynamic mathematics software package designed and developed for the secondary school mathematics education in China. In terms of mathematical content, it covers geometry, algebra, calculus, probability, statistics, and so on. We would say it is:

Dynamic geometry software + Computer algebra system + Dynamic simulation platform + Programming environment + Geometry theorem Prover + ...

The following are some of SSP's possibilities:

(1) With the intelligent SketchPen tool of SSP, we can construct most of the geometric objects directly just like drawing them on blackboard using chalk. For example, in Figure 8, AB is perpendicular to CD as diameters of circle C, Point F is on circle C, FG is perpendicular to AB, FH is perpendicular to DE. The graph can be plotted quickly using SketchPen only:

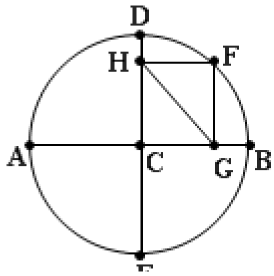

Figure 8: *the modle of a ladder*

(2) The coordinate points and the graph of an equation with parameters can be plotted directly, and the parameters will be created automatically. For example, we can directly construct the coordinate points A (5*cos(t),0) and B (0,5*sin(t)), and then drag any of them easily. The curve given by the following parametric function can also be plotted through inputting its equation:

$$\begin{cases} x=a\cdot\cos^3(t) \\ y=a\cdot\sin^3(t) \end{cases} \quad t\in[0,\ 2\pi]$$

(3) A free point on a line, a circle, a conic section or another curve is distributed by a variable, so we can move it to any position on the line or curve accurately by changing the value of the variable using an animation button. For instance, in Figure 9, point B is on circle O. We can move the point through an animation button to make the value of angle AOB to be π/6, π /3, π /4, 1, 2.1, or any other value.

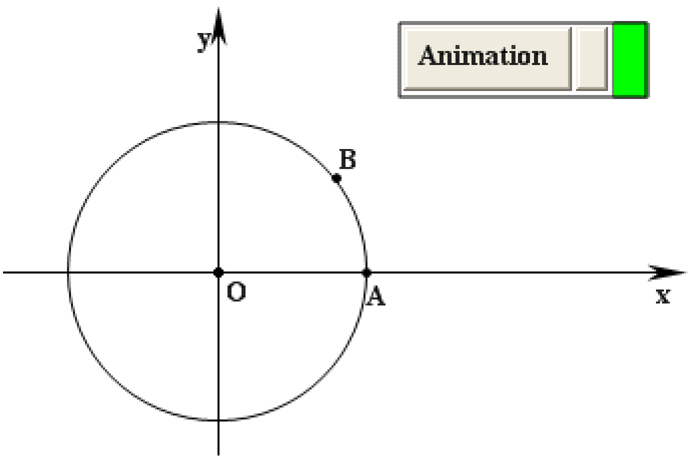

Figure 9: *point on a circle*

(4) A conic section can be plotted directly with SSP in more than ten ways. For example, we can construct the ellipse passing point C with foci point A and point B through clicking the

corresponding order from the Construct menu.

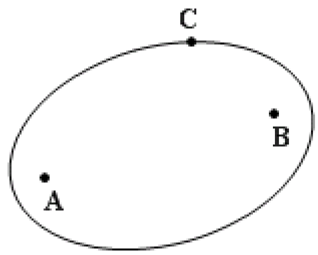

Figure 10: *ellipse passing a point with two foci*

(5) In SSP we can construct the locus of a point determined by more than one movable points.

The following is an example: Points C, F, and I are respectively on circle A, circle D, and circle **G.** Point J is on segment CF and point K is on segment IJ, shown in Figure 11. What is the locus of point K as points C, F, and I move on their own circles respectively with different ways?

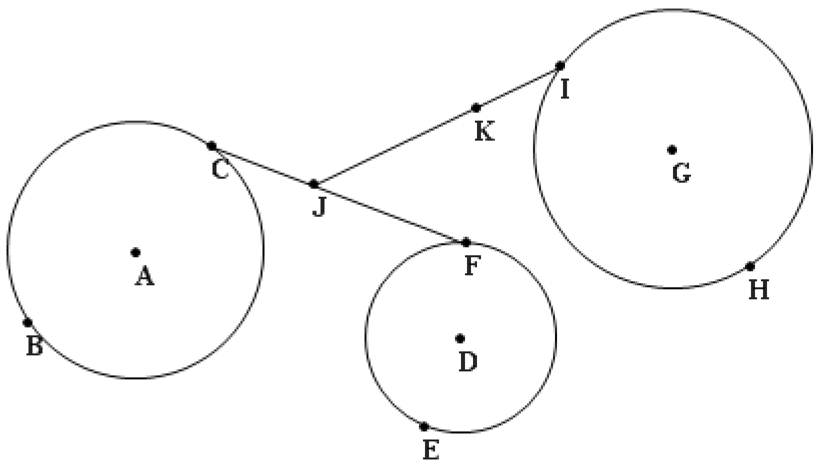

Figure 11: *three point on three circle respectively*

The graphics in Figure 11 show some of the locus curves of point K constructed in SSP.

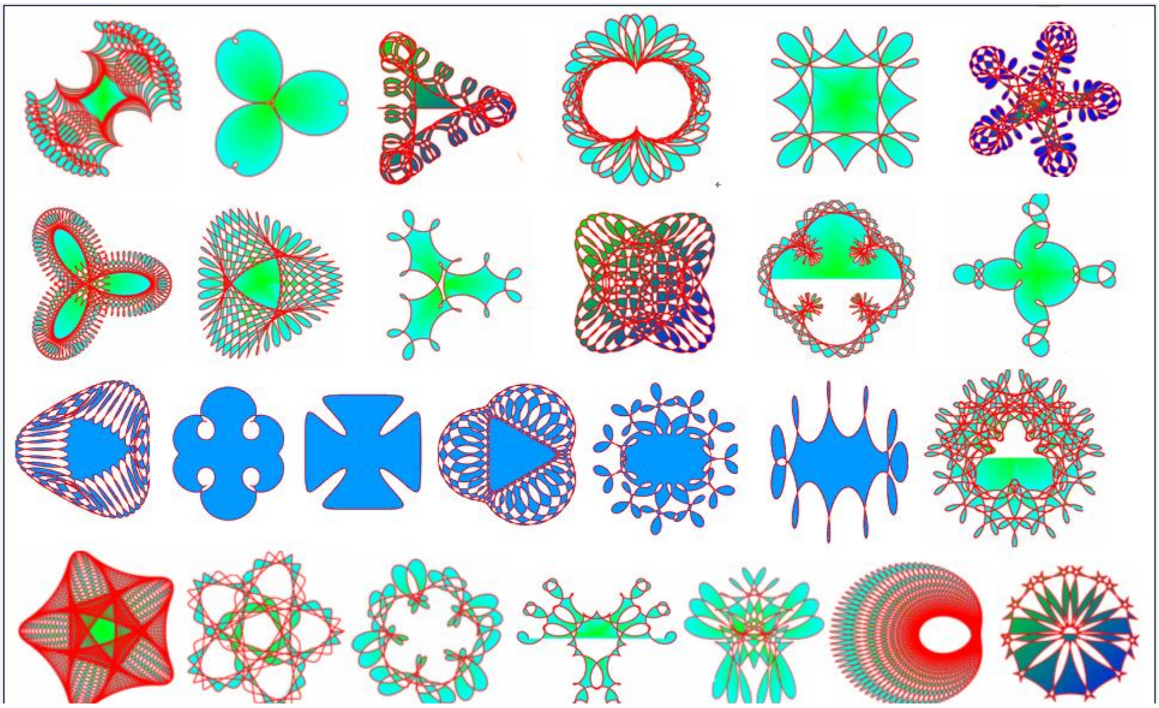

Figure 12: *the loci of point K*

(6) SSP is a geometry theorem prover.

The following is a problem from a mathematics textbook of grade 10 (see [9]):

Point O is the intersection of the two diagonals of parallelogram ABCD. AE is perpendicular to BD， CF is perpendicular to BD, and point E and F are on segment BD. Show that AE = CF.

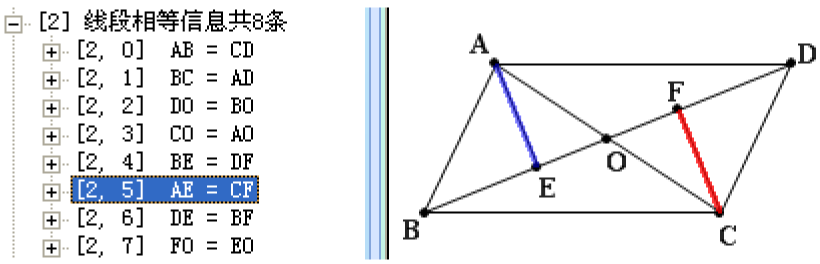

Figure 13: *showing AE=CF*

Construct the graph with SSP, and then click Auto Proving from the Prove menu. About two seconds later, the computer will finish the proving process. The proved conclusion AE=CF can be found in the Proved Information Workshop, shown in Figure 13.

How did SSP prove AE=CF? Click the symbol +, and we can find the conditions which deduce the conclusion, shown in Figure 14.

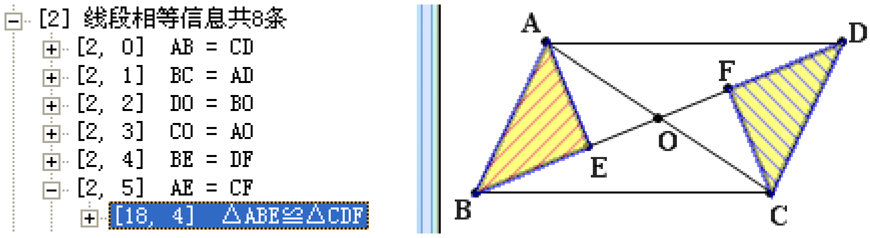

Figure 14: *showing the two congruent triangles*

Then you can continue to click the symbol + in the front of the information to see how the conclusion is deduced (see Figures 15 and 16).

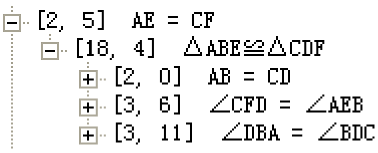

Figure 15: *decucing*

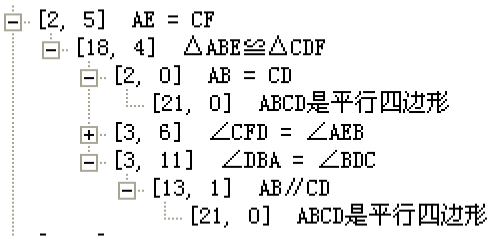

Figure 16:*decucing*

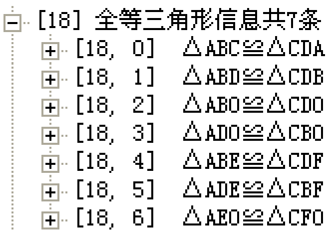

Figure 17: *pairs of congruent triangles*

SSP proves with the geometric relationships created during plotting and the rules that are the basic theorems and concepts from the secondary school geometry textbooks. It can deduce several kinds of information, such as Equal Angles, Perpendicular lines, Parallel lines, Congruent Triangles, Similar Triangles, and so on. For example, there are seven sub-conclusions about congruent triangles deduced by SSP, shown in Figure 17.

In addition, SSP permits users to reset the rules by themselves, and then it automatically proves with these rules and the related geometric relationships.

(7) The SSP Algebra Workshop is also a programming environment. We can learn about it through

solving the following problem: How many intersection points are there between the graph of y=a^x and the graph of its reverse function?

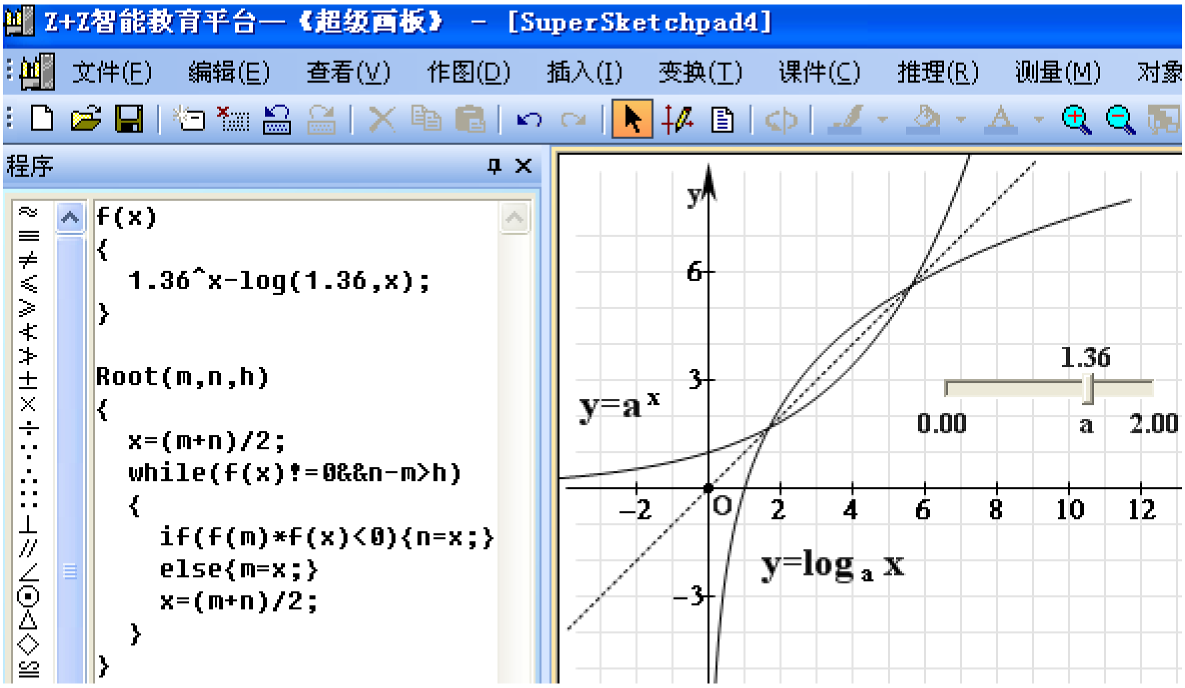

Figure 18: *Drawing workshop and Programming environment*

The order f(x){1.36^x-log(1.36,x)} is to define the function f(x) as f(x)=1.36^x-log(1.36,x). Root(a,b,h) is the function to find the approximate solution of f(x) at [a,b] with dichotomy, where h represents the degree of precision of the solution. By observing the graph in Figure 18, we can see that the two solutions are in intervals [1, 2] and [4, 8] respectively. After running the orders Root(1,2,0.001) and Root(4,8,0.001), two approximate solutions with the degree of precision 0.001 will come out.

(8) In SSP we can make the computer to simulate probability and random experiments like tossing coins and rolling dices. Figure 19 shows the Monte Carlo Simulation developed with SSP.

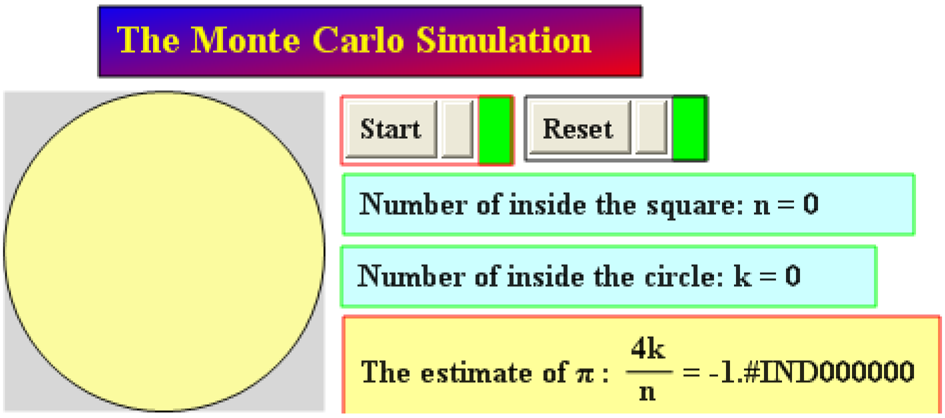

Figure 19: *the Monte Carlo Simulation*

Click the Start button, and 1000 trials will occur. At the same time, the number of trials, the number of the points inside the circle, and the estimated value of π are counted/calculated automatically. The experimental process and result are shown respectively in Figures 20 and 21.

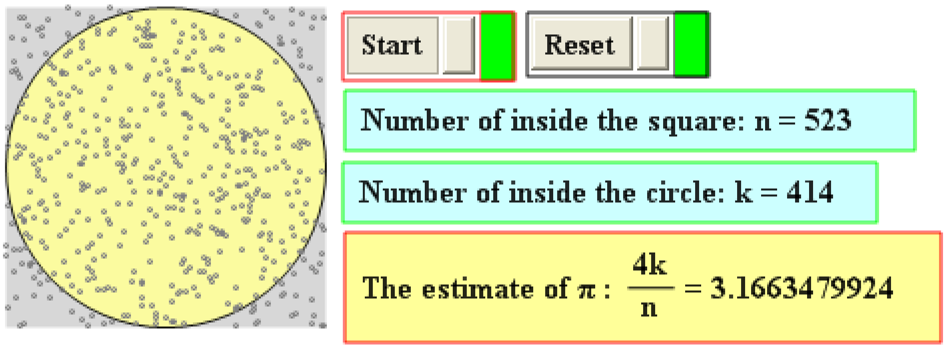

Figure 20: *the process of experiment*

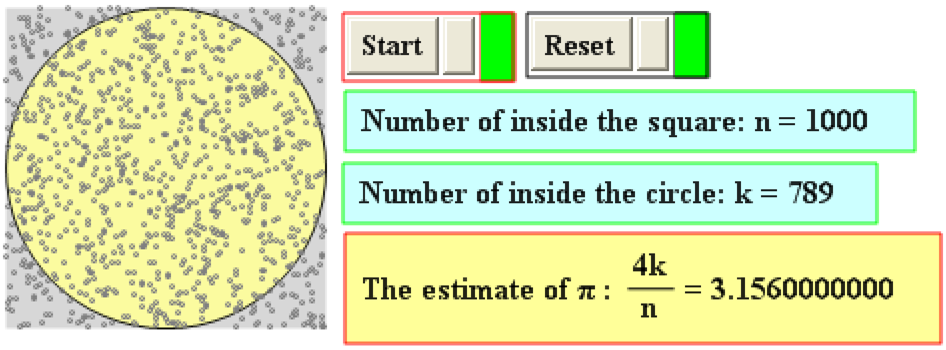

Figure 21: *the result of experiment*

Another 1000 random trials will occur by clicking on the Start button again. Teachers and students can reset the number of trials through the dialog boxes of the Start button. For example, if you change 1000 into 2000, 2000 trials will occur after clicking the Start button.

3.3 The Use of SSP in China

In order to help teachers to use IT more efficiently in their mathematics teaching, the Chinese Ministry of Education launched a project entitled “The Experimental Research on Using SuperSketchpad in the Reform of the National Mathematics Curriculum” in 2003. Since then, SSP has been integrated with most of the new mathematics curricula for creating mathematics learning activities. The committee of this project consists of mathematics teachers, mathematics educators, mathematicians, textbook developers and IT experts.

This project attracted 117 schools in its first two years. Up to now, more than one thousand schools have been using SSP in math education. Book [10] and book [11] have gathered plenty of materials about teaching and learning mathematics using SSP.

There are many comments of teachers using SSP in book[10]. The following are some of them:

★"Learning with SSP, students' abilities to explore and discover mathematics have been developed obviously. Students gradually love mathematics since they are fascinated by SSP, and they prefer learning and creating mathematics to playing on-line games during holidays and/or weekends. At the same time, teachers are promoted to know more about mathematics to be ready for students' various kinds of novel questions." said Wang Mingyu, a teacher from the Affiliated School of Beijing University.

★"SSP helps us attaining lots of what can not be realized with traditional methods in the past, and teachers' ideas of mathematics teaching have been enriched greatly." said Cheng Kai, a teacher from Yichang No. 9 Middle School, Hubei Province.

★ "Using SSP in mathematics education has a great effect on the teaching and learning of mathematics. It takes about 3 minutes or less to develop a courseware unit using SSP. So teachers can construct the mathematics materials even in class. This helps create a better learning environment for students, and the gap between students and the teacher (or curriculum) has been narrowed." said Zeng Meilu and Zhou Shengwei, teachers from the Center of Mathematics Teaching and Research in Jinan.

SSP has effects on saving teachers’ time and reducing teachers’ workload so it becomes possible for teachers to use mathematics education software in their daily activities. When teachers benefit from IT, they will be more willing to explore and discover the value of IT in mathematics education.

3.4 The Future Work on SSP

As the first dynamic mathematics software package designed for mathematics education of secondary school in China, SSP has many aspects to be advanced and to be perfect in the future. The following are some of them:

(1) When dragging the point constructed by coordinates (cos(t), sin(t)), the coordinate point cannot be controlled well in direction or position.

(2) The corresponding curve can be plotted directly through entering its parametric equation y=x^2+(2m+1)*x+m^2-1, but as *m* varies, the locus of the vertex or focus of this parabola cannot be constructed directly.

(3) SSP cannot plot the intersection points of two conic sections directly, while such construction is usually needed.

(4) The direct construction of a parabola through three points is not available in SSP, which is also frequently needed.

4. Conclusion

It is important to introduce excellent education software from other countries or regions, but it might be more important to design software that addresses the limitations of the existing software and better meets the various needs of the teachers in the country or region.

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